

Typical solution of the final exam in Time Series Analysis module

Exercise 01 (7pts): choose the correct answer

1. Assume that you observed the daily oil prices and the daily US Dollar exchange rates over the period from 2000 to 2023. The kind of data you obtained is **Time series data (0.5)**
 - Panel data - Cross-sectional data - Time series data - Pooled data
2. Suppose you study the impact of the interest rates (r_t) on investment (I_t) in the economy of a particular country. The dependent variable is **Investment (0.5)**
 - Interest rates - Investment
3. If the value of the SST is 3400 and SSE =1000, then the R^2 is?
 - ☐ 29.41%
 - ☐ 34 %.
 - ☒ 70.58% **(0.25)**

Explanation: $R^2 = 1 - \frac{SSE}{SST} = 1 - (1000/3400) = 70.58$ **(0.5)**
4. If the value SST= 4200 and SSE = 1400, then the explained variation of dependent variable is?
 - ☐ 1400
 - ☐ 2800 **(0.25)**
 - ☒ 66.67% **(0.25)**

Explanation : explained variation is SSR, $SSR = SST - SSE = 4200 - 1400 = 2800$ **(0.5)**

Explained variation = $2800/4200 = 66.67\%$ (0.5)
7. If the R^2 is 80% this means that?
 - ☐ The dependent variable explained 80% of the variation of the independent variable.
 - ☒ 80% increase or decrease in dependent variable value is explained by the independent variable. **(0.25)**
 - ☐ 20 % of the variation in the dependent variable value left unexplained by the chosen independent variable. **(0.5)**
8. To perform multiple linear regression, the assumption of **Homoskedasticity in the data (0.5)** must be met.
 - ☐ Heteroskedasticity in the data.

- Multicollinearity

- Homoskedasticity in the data

9. The serial correlation means

- Correlation of a time series with its own past or future values (0.5)

- Autocorrelation (0.5)

- Correlation between dependent and independent variable

10. Multicollinearity refers to the.....

- The presence of high correlations among dependent variable and independent variables

- The presence of high correlations among independent variables in a regression model (0.5)

- The presence of the autocorrelation in the independent variables.

11. To compare the explanatory power of two models we examine the adjusted R-squared value and f-statistic.

- True (0.5) - False

12. The null hypothesis in the context of the F-test is that all coefficients of the independent variables in the model are equal to zero.

- False - True (0.5)

Questions 02 (13pts):

Consider that you examined the effect of US interest rates (r_t) and US Dollar exchange rates (er_t) on oil prices (OP_t), and the results you obtained are as follows :

Variable	Coefficient	Std. Error	t-Statistic
Const	98.67	16.45	5.998(0.25)
r_t	- 1.875	0. 793	- 2.364(0.25)
er_t	-3.651	0.9653	- 3.782(0.25)
R-Squared 82.6		F-Stat 142.5 prob (f-stat) = 0.00001	
Adjusted R-Squared 82.08		SSE 200	
Observations 63		SSR 950	
		SST 1150	

First, we calculate the t-statistic: $t\text{-stat} = \text{coefficient} / \text{std. Error} (0.25) = 98.67/16.45 = 5.998$, $-1.875/0.793 = -2.364$, $-3.651/0.9653 = -3.782$

Second, calculate SST: $SSE = SST - SSR (0.25) = 1150 - 950 = 200 (0.5)$

Third, R-Squared = $SSR/SST = 950/1150 = 82.6\% (0.5)$

Adjusted R-Squared = $1 - (((n-1)/(n-k-1))(1-R^2)) (0.25) = 0.8208 = 82.08\% (0.5)$

f- statistic: $F = \frac{\frac{SST-SSE}{k}}{\frac{SSE}{n-k-1}} (0.25) = ((950/2)/(200/60)) = 142.5 (0.5)$

- Write the regression equation that showing the relationship between variables under the study.

According to the exercise data, it is evident that the dependent variable is the Oil prices (OP_t) and the independent variable is the US interest rates (r_t) and US Dollar exchange rate (er_t), the regression equation is as follow **(0. 25)**:

$$OP_t = \beta_0 + \beta_1 r_t + \beta_2 er_t + \varepsilon \quad \textbf{(0. 5)}$$

$$OP_t = 98.76 - 1.875r_t - 3.561er_t \quad \textbf{(1)}$$

- the study hypothesis regarding the relationship between the independent variables and the dependent variable are as follow:

the equation is

$$OP_t = \beta_0 + \beta_1 r_t + \beta_2 er_t + \varepsilon$$

The hypothesis $\begin{cases} H_0: \beta_1 = 0 \\ H_1: \beta_1 \neq 0 \end{cases}$

$\begin{cases} H_0: \text{there is no relationship between oil prices and interest rates} \\ H_1: \text{there is a relationship between oil prices and interest rates} \end{cases} \quad \textbf{(0. 5)}$

The hypothesis $\begin{cases} H_0: \beta_2 = 0 \\ H_1: \beta_2 \neq 0 \end{cases}$

$\begin{cases} H_0: \text{there is no relationship between oil prices and exchange rates} \\ H_1: \text{there is a relationship between oil prices and exchange rates} \end{cases} \quad \textbf{(0. 5)}$

- Test the hypothesis at 1% level of significance, and give the interpretation of the relationship between oil prices and the US interest rates and US Dollar exchange rates.

$$OP_t = \beta_0 + \beta_1 r_t + \beta_2 er_t + \varepsilon$$

$$OP_t = 98.76 - 1.875r_t - 3.561er_t$$

To test the hypothesis and interpret the results, we have to compare the t-statistic calculated to the t-statistic critical (value from the table of t-distribution)

The first variable US interest rates: $t - statistic_{cal} \text{ for } \beta_1 = 2.364$

$t - statistic_{crit} = 2.390$ from the table we have $\alpha = 0.01$ and $df = n - k - 1 = 63 - (2 + 1) = 60$ **(0. 5)**

Since $t - statistic_{cal} < t - statistic_{crit}$ so, we conclude that the value of β_1 is insignificant we reject the alternative hypothesis and we accept the null hypotheses **(1)**; this means that the US interest rates (r_t) has no effect on oil prices **(1)**.

The second independent variable: $t - statistic_{cal}$ for $\beta_2 = -3.782$

$$t - statistic_{crit} = 2.390$$

Since $t - statistic_{cal} > t - statistic_{crit}$ so, we conclude that the value of β_2 is significant we reject the null hypothesis and we accept the alternative hypotheses **(1)**; this means that the US Dollar exchange rate (er_t) has a significant and negative effect on the Oil Prices (OP_t) **(1)**, one unit increase (decrease) in US Dollar exchange rates the Oil prices Decrease (increase) by 3.782 unit **(1)**

5. Do the two independent variables explain well the variation of the dependent variable?

Since the two variables together explain 82.6% of the variation of the dependent variable and the value of the F-statistic is significant this means that the two independent variables explain well the variation of the dependent variables **(1)**.